Abstracts for the *Iberian meeting on numerical semigroups - Vila Real 2012*

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Abdallah Assi *Université d'Angers*

Constructing the set of complete intersection numerical semigroups with a fixed Frobenius number

Delorme suggested that the set of all complete intersection numerical semigroups can be computed recursively. We have implemented this algorithm, and particularized it to several subfamilies of this class of numerical semigroups: free and telescopic numerical semigroups, and numerical semigroups associated to an irreducible plane curve singularity. The recursive nature of this procedure allows us to give bounds for the embedding dimension and for the minimal generators of a semigroup in any of these families.
This is a joint work with P. A. García Sánchez.

Valentina Barucci *Università di Roma “La Sapienza”*

Differential operators of numerical semigroup rings

The subject of the talk is the ring of differential operators on $\mathbb{C}[S]=\mathbb{C}[t^{d_1},\dots,t^{d_\nu}]$, where $S=\langle d_1,\dots,d_\nu\rangle$ is a numerical semigroup. It is not a commutative ring, but its associated graded in the filtration induced by the order of the differential operators is a commutative ring and it is particularly easy to describe it in case S is an Arf semigroup.
These results are contained in a joint paper with Ralf Fröberg.

Manuel B. Branco *Universidade de Évora*

Two-extension of a numerical semigroup with embedding dimension two

Let S and $\overline{S}$ be two numerical semigroups such that $S\subseteq\overline{S}$ and let d be a positive integer. We say that $\overline{S}$ is a d-extension of S if

$$d\overline{S}=\{dx\ :\ x\in\overline{S}\}\subset S.$$

Let n_1 and n_2 be two integers greater than or equal to 2 such that $\gcd(n_1,n_2)=1$. We denote by $\mathcal{F}(n_1,n_2)$ the set of numerical semigroups S such that S is a 2-extension of $\langle n_1,n_2\rangle$.
In this talk we will present a study of the set $\mathcal{F}(n_1,n_2)$. Given a rational number q we denote by $\lfloor q\rfloor$ the integer $\max\{z\in\mathbb{Z}\ :\ z\leq q\}$. First we will see that all element of $\mathcal{F}(n_1,n_2)$ is fully determined by a subset of incomparable elements of $\mathcal{B}(n_1,n_2)=\left\{1,\dots,\lfloor\frac{n_2}{2}\rfloor\right\}\times\left\{1,\dots,\lfloor\frac{n_1}{2}\rfloor\right\}$. Using [5] we obtain formulas for the Frobenius number and the genus for a numerical semigroup in $\mathcal{F}(n_1,n_2)$ as a function of the subset of incomparable elements of $\mathcal{B}(n_1,n_2)$ that determines it.
We will also show how we can compute all numerical semigroups in $\mathcal{F}(n_1,n_2)$ for one determinate embedding dimension. As a consequence we obtain formulas for the cardinality of the sets $\mathcal{F}(n_1,n_2)$ and $\{S\in\mathcal{F}(n_1,n_2)\ :\ e(S)=n\}$.
This is a joint work with J. C. Rosales.

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Maria Bras-Amorós *Universitat Rovira i Virgili*

On the maximal gap of an ideal and the Feng-Rao numbers

The maximum gap of a numerical semigroup is usually referred to as the Frobenius number of the semigroup. By the pigeonhole principle it is easy to prove that the Frobenius number is at most twice the genus minus one, and there are semigroups attaining this bound (called symmetric semigroups). An ideal of a numerical semigroup is a subset of the semigroup such that any element in the subset plus any element of the semigroup add up to an element of the subset. The ideal will be a subset of $\mathbb{N}_0$ with finite complement in it. The elements in this complement are called gaps of the ideal. Our first result is an analogous of the upper bound on the Frobenius number for the largest gap of an ideal.
Given a linear code it is very important not only in coding theory but also in cryptography to study the so-called generalized Hamming weights of the code (the minimum size of the support of subspaces of any given dimension) [6, 5]. A nice tool for takling these weights for AG codes are the generalized order bounds introduced in [4], involving Weierstrass semigroups. In [2], a constant depending only on the semigroup and the dimension of the Hamming weights was introduced, from which the order bounds could be completely determined for codes of rate low enough. This constant was called Feng-Rao number in the same reference. Some results related to the Feng-Rao numbers can be found in [3, 1]. In the present contribution, using the upper bound on the maximum gap of an ideal, we derive a lower bound on the so-called Feng-Rao numbers and so a new bound on the Hamming weights.

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Lance Bryant *Shippensburg University*

Filtrations in One-Dimensional Analytically Irreducible Noetherian Local Domains

Let R be a one-dimensional analytically irreducible Noetherian local domain having the same residue field as its integral closure. We will discuss using the corresponding numerical semigroup to determine if the associated graded ring of R with respect to a filtration is Cohen-Macaulay or Gorenstein. Much attention has been given to the associated graded ring of R with respect to the maximal ideal in this context, and we will provide results regarding other filtrations including I-adic and integral closure filtrations.

Jonathan Chappelon *Université Montpellier 2*

On the Möbius function for arithmetic numerical semigroups

Let S be a numerical semigroup, that is, a subset of $\mathbb{N}$ containing 0, stable by addition and whose complementary part is finite in $\mathbb{N}$. Let P_S be the poset structure induced by S on $\mathbb{N}$, that is, for any natural integers x and y,

$$x\leq_{P_S}y\quad\Longleftrightarrow\quad y-x\in S.$$

Here, we are interested in studying the Möbius function μ_S associated to the poset $(\mathbb{N},\leq_{P_S})$, that is, the function defined by, for all natural integers x and y,

$$\mu_S(x,y)=\sum_{j\geq 0}(-1)^j c_j(x,y),$$

where $c_j(x,y)$ represents the number of chains of length j between x and y. In 1979, Deddens gave an explicit formula for μ_S when $S=\langle a,b\rangle$. In this talk, we shall discuss μ_S when $S=\langle a,a+d,\dots,a+kd\rangle$ is an arithmetic sequence. We first present a new direct proof of Deddens' result (our proof is shorter than the original one based on a recursive case-by-case analysis). By using this new approach, we are able to obtain a recursive formula for μ_S, in the general case. We use the latter to give an explicit formula in the case when $k=2$ and a is even. Finally, we will discuss the difficulty for the case $k=2$ and a odd. This is a joint work with J. Ramírez Alfonsín.

Scott T. Chapman *Sam Houston State University*

Delta Sets of Numerical Monoids Using Non-Minimal Sets of Generators

Several recent papers have studied the structure of the delta set of a numerical monoid. We continue this work with the assumption that the generating set S chosen for the numerical monoid M is not necessarily minimal. We show that for certain choices of S, the resulting delta set can be made (in terms of cardinality) arbitrarily large or small. We close with a close analysis of the case where $M=\langle n_1,n_2,in_1+jn_2\rangle$ for nonnegative i and j.

Teresa Cortadellas *Universitat de Barcelona*

On the Apéry sets of monomial curves

By using an idea of Valentina Barucci and Ralf Fröberg about the existence of Apery basis, we may attach to a numerical semigroup ring a table, that we call the Apery table, which consists on the Apery sets with respect to the multiplicity of the successive powers of the maximal ideal up to the reduction number. This table reflects many properties of the tangent cone of the corresponding semigroup ring, as we have showed in previous work. The proposal of the talk is to provide some new applications to several problems. Among other results we may prove, for instance, that the Hilbert function of a 4 generated numerical semigroup ring whose tangent cone is Buchsbaun is non decreasing.
This is a joint work with Raheleh Jafari and Santiago Zarzuela.

Marco D'Anna *Universitá di Catania*

The Apéry set and the associated graded ring of a numerical semigroup ring

I will show how the properties of the Apéry set of a numerical semigroup reflect the properties of the associated graded ring of the corresponding semigroup ring: in particular, I will focus on a new class of numerical semigroups that allows to give a characterization of the Complete Intersection property. Moreover, I will introduce two more classes of numerical semigroups that yields naturally in this context and I will show the relations with other known classes of numerical semigroups.

Shalom Eliahou *Université du Littoral Côte d'Opale*

On numerical semigroups $\langle a,b\rangle$ of prime power genus

Given an integer g, how many numerical semigroups $\langle a,b\rangle$ with exactly g gaps are there? Let $n(g,2)$ denote their number. A theorem of Sylvester implies that $n(g,2)$ actually counts certain special factorizations of $2g$. In this talk, we first determine $n(2^k,2)$ for all k. Then for p an odd prime, we determine $n(p^k,2)$ for $k\leq 8$, and show that its value for $k=4097$ depends on the as yet unknown factorization of the 12th Fermat number $2^{2^{12}}+1$. Finally, we show that for arbitrary k, the value of $n(p^k,2)$ only depends on the class of p modulo some explicit integer $M(k)$.
This is joint work with Jorge Ramírez Alfonsín.

Ralf Fröberg *Stockholms Universitet*

On the degree of syzygies for semigroup rings

Leonid Fel proved a set of identities for the Betti numbers of a semigroup ring. We will, using some commutative algebra, give elementary proofs of these.

J. I. García-García *Universidad de Cádiz*

Affine convex body semigroups

Let F be a subset of $\mathbb{R}^k$, $\mathbf{F}=\bigcup_{i=0}^{\infty}F_i\cap\mathbb{R}^k_{\geq}$ and $\mathcal{F}=\bigcup_{i=0}^{\infty}F_i\cap\mathbb{N}^k$, where $F_i=\{iX\mid X\in F\}$ with $i\in\mathbb{N}$. A convex body of $\mathbb{R}^n$ is a compact convex subset with non-empty interior (with the usual topology of $\mathbb{R}^n$). If F is a convex body, then the set $\mathbf{F}$ is a monoid and $\mathcal{F}$ is a semigroup. Given a convex body F, we call the convex body monoid (respectively semigroup) generated by F to the above monoid (respectively semigroup) $\mathbf{F}$ (respectively $\mathcal{F}$). These semigroups generalizes the concept of proportionally modular numerical semigroup but they are not necessarily finitely generated.
If $\mathcal{F}$ is a finitely generated semigroup we say that $\mathcal{F}$ is an affine convex body semigroup. Given a convex polygon or a circle in $\mathbb{R}^2$, we study necessary and sufficient conditions for $\mathcal{F}$ to be finitely generated. These conditions are related to the slopes of the extremal rays of the minimal cone which includes to $\mathcal{F}$. We give effective methods to obtain their minimal system of generators and we generalizes the concept of modular Diophantine inequality for convex body monoids.
All the implementations of the algorithms of this work are available at the url `http://www.uca.es/dpto/C101/pags-personales/alberto.vigneron`.
This is a joint work with María Ángeles Moreno-Frías, Alberto Vigneron Tenorio, and Alfredo Sánchez-R.-Navarro.
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Micah Leamer *Chennai Mathematical Institute*

Torsion and tensor products over numerical semigroup hypersurfaces

Over a generalized hypersuface domain the tensor product of two non-free modules always has torsion. We give lower bounds for the length of the torsion submodule of I tensor J where I and J are monomial ideals over a numerical semigroup hypersurface. We also give a lower bound on the number of generators of $I\otimes\mathrm{Hom}(R,I)$ where R is a one-dimensional analytically irreducible residually rational hypersurface and I is any ideal. The integral closure of a one-dimensional analytically irreducible ring is a discrete valuation ring. In the process of proving these results we show how the tensor product of two ideals over an analytically irreducible residually rational ring relates to the tensor of the respective ideals over the numerical semigroup determined by the valuation.

David Llena *Universidad de Almería*

On the Feng-Rao numbers for numerical semigroups with two generators

The generalized order bounds for one-point AG codes are described in terms of a certain Weierstrass semigroup. The asymptotical behaviour of such bounds differs from that of the classical Feng-Rao distance, for $r\geq 2$, by the so-called Feng-Rao numbers. In fact, all these parameters are invariants that make sense for arbitrary numerical semigroups, and not only for Weierstrass semigroups or in applications in coding theory. This paper is addressed to compute the Feng-Rao numbers for numerical semigroups of embedding dimension two (with two generators), for which there seems to exists a closed simple formula.
This is a joint work with Manuel Delgado, J.I. Farrán and P.A. García-Sánchez.

Carlos Marijuán *Universidad de Valladolid*

Classification of 3-numerical semigroups by means of L-shaped tiles

A g-numerical semigroup has associated a monomial curve which can be seen in the affine space $\mathbf{A}_g$. We consider some classes of numerical semigroups useful in the study of curve singularity. These semigroups receive the name of the corresponding classes of associated curves.
On the other hand, it is known that each 3-numerical semigroup has associated one or two periodic plane tessellations by means of L-shaped tiles. Some combinatoric properties of the semigroup can be described in terms of the parameters of the tiles.
In this work we give a characterization of the parameters of the L-shaped tiles associated to a 3-numerical semigroup in terms of its generators and we use it to classify the 3-numerical semigroups of interest in Geometry. This is a joint work with Francesc Aguilo.
This work has been supported by the projects MTM2011-28800-C02-01, 2009SGR1387, and MTM2007-64704.

Ivan Martino *Stockholms Universitet*

Random Walk on Semigroup

It is a well known fact that the set of functions over a (topological) space X covers its topology. We would like to understand properties of a semigroup via some properties of its set of functions.
Let S be a semigroup (not necessarily numerical) and let $S^{(gr)}$ be its group completion, that is defined by adding to S the inverse elements. Let us assume that $S^{(gr)}$ is compact.
We can consider $L^2(S^{(gr)})$, the set of L^2-measurable functions on $S^{(gr)}$. We state the necessary and sufficient condition for $f\in L^2(S^{(gr)})$ lying in $f\in L^2(S)$. This condition are in term of integral equations. If we narrow down our investigation to the semigroup $S\subseteq\mathbb{N}^r$, then this integral equations can be seen as a 'suitable' Fourier analysis constrains.
Finally for the numerical semigroup $S\subseteq\mathbb{N}$ (and $S^{(gr)}=\mathbb{Z}$), we study some stochastic process over S. The generating function of a random walk over a numerical semigroup lies in $L^2(S)$. The study of these functions says some information about the topology that S assume under our assumption.
This is a joint work with Luca Martino (Universitad Carlos III, Madrid, Spain).

Vincenzo Micale *Università di Catania*

On the Betti numbers of some semigroup rings

For any numerical semigroup S, there are infinitely many numerical symmetric semigroups T such that $S=\frac{T}{2}$ (for the definition of $\frac{T}{2}$ see below) is their half. We are studying the Betti numbers of the numerical semigroup ring $K[T]$ when S is a 3-generated numerical semigroup or telescopic. We also consider 4-generated symmetric semigroups and the so called 4-irreducible numerical semigroups.

Julio J. Moyano-Fernandez *Osnabrueck University*

Numerical semigroups and Hilbert series of graded modules

Let F be a field, and consider the positively Z-graded polynomial ring $R=F[X,Y]$. In this talk we will briefly discuss the role played by numerical semigroups in the characterization of Hilbert series of finitely generated graded R-modules of positive depth.

Ignacio Ojeda *Universidad de Extremadura*

Homogeneus catenary degree

In this talk, we will introduce the concept of homogenous catenary degree of a positive affine semigroup S as the catenary degree of the homogeneization S^h of S. We will prove that the catenary degree is upper bounded by the homogenous catenary degree. In particular, if the semigroup S is homogeneous (i.e., $S^h=S$), we will see that for each $m\in S$, there exists $b\in\mathrm{Betti}(S)$, such that $c(m)=c(b)$; this actually means that the catenary degrees of the elements in a homogeneous affine semigropup S are concentrated in the total degrees of a minimal system of S-graded generators of the lattice ideal associated to S. In the talk, we will explore the behavior of other factorization properties under homogeneization.
This is a joint work with P.A. García Sánchez.

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Gilvan Oliveira *DMAT/UFES/BR*

Weierstrass Semigroups of a Recursive Optimal Tower of Function Fields

It is well known that algebraic function fields over finite fields have many applications in coding theory and cryptography. The so-called algebraic geometric codes are defined by using Riemann-Roch spaces of divisors on function fields over finite fields. One can obtain codes having parameters that attain the Tsfasman-Vladut-Zink bound by constructing recursive tower of function fields.
Our purpose here is to give an algorithm to compute bases for Riemann-Roch spaces and Weierstrass semigroups over the recursive optimal tower of function fields defined below. As an application we also explicitly present Weierstrass semigroups till level eight.
The object of study is the tower over the finite field $K=\mathbb{F}_{p^2}$ with p^2 elements, where p is an odd prime, defined recursively by: $T_0=K(x_0)$ and, for $j\geq 0$, $T_{j+1}=T_j(x_{j+1})$, where the function x_{j+1} satisfies the relation: $x_{j+1}^2=(1+x_j^2)/2x_j$. This tower was introduced and proven to be optimal by A. Garcia and H. Stichtenoth. By denoting $g(T_j)$ the genus of T_j and $N(T_j)$ the number of its $\mathbb{F}_{p^2}$-rational points, it means that the limit of $N(T_j)/g(T_j)$ as $j\to\infty$ attains the Drinfeld-Vladut upper bound $p-1$.
Let P_∞^j be the unique pole of the function x_0 in T_j. For each $s\in\mathbb{N}$, fix the divisors sP_∞^j and let $L(sP_\infty^j)$ be the Riemann-Roch space defined by: $L(sP_\infty^j)=\{x\in T_j\mid\ \text{the divior of } z \text{ satisfies } (z)\geq -sP_\infty^j\}$.
The main result is an algorithm to compute bases for such linear spaces. The central idea to do this is to apply results of H. Maharaj to decompose the vector space $L(sP_\infty^j)$ in T_j as a direct sum of Riemann-Roch spaces of divisors at the lower level T_{j-1}, and continue this way till the rational function field T_0, where the bases can be easily computed.
In order this process to be performed, the divisors we get at each level $k<j$ should be invariant for the action of the Galois group of T_k/T_{k-1}. Unfortunately this condition is not always satisfied and the procedure has to be suitably modified. As a consequence of the main result we get an algorithm to compute the Weierstrass semigroups: $H(P_\infty^j)=\{s\in\mathbb{N}\mid\exists z\in T_j\ \text{s.t.}\ (z)_\infty=sP_\infty^j\}$.
This is a joint work with with Francesco Noseda and Luciane Quoos.

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Jorge Ramírez-Alfonsín *Université Montpellier 2*

Pseudo-symmetry for almost arithmetic semigroups

A numerical almost arithmetic semigroup (AA-semigroup) consists of all non-negative integer linear combinations of relatively prime positive integers $a,a+d,a+2d,\dots,a+kd,c$ where also a,d,k,c are positive integers. A semigroup S is said to be pseudo-symmetric if the Frobenius number $f(S)$ is even and $S\cup(f-S)=Z\setminus f/2$. In this talk we give a characterization of pseudo-symmetric AA-semigroups. We also present some results about the structure of irreducible AA-semigroups which can be used to construct classes of symmetric AA-semigroups and of pseudo-symmetric AA-semigroups. This is a joint work with O.Rodseth.

Aureliano M. Robles-Pérez *Universidad de Granada*

Some questions on numerical semigroups with type two

Let S be a numerical semigroup. Following [1], we denote the *genus*, the *set of pseudo-Frobenius numbers*, and the *Frobenius number* of S by $g(S)$, $\mathrm{PF}(S)$, and $F(S)$, respectively. Moreover, the *type* of S is the cardinality of $\mathrm{PF}(S)$.
The numerical semigroups with type one are called *symmetric* (see Corollary 4.11 in [1]). For this class of numerical semigroups we have the following result.
Lemma[1, Corollary 4.5] S *is a symmetric numerical semigroup if and only if* $g(S)=\frac{F(S)+1}{2}$.
Two easy consequences of this lemma are collected in the next proposition.
Proposition
1. $\{F\}$ *is the set of pseudo-Frobenius numbers of a numerical semigroup with type one if and only if* F *is an odd integer greater than or equal to* -1.
2. If two numerical semigroups of type one have the same Frobenius number, then they have the same genus.
At this point, some questions arise naturally.
1. What conditions must satisfy two integers g_1,g_2 in order to there exists a numerical semigroup S such that $\mathrm{PF}(S)=\{g_1,g_2\}$?
2. If two numerical semigroups of type two have the same Frobenius number, what are their genera?
3. If two numerical semigroups of type two have the same set of pseudo-Frobenius numbers, what are their genera?

Our purpose is to answer these questions.

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Alessio Sammartano *Universitá di Catania*

Wilf's conjecture for numerical semigroups

Let S be a numerical semigroup with Frobenius number $f(S)$, embedding dimension $\nu(S)$ and let $n(S)=\mid S\cap[0,f(S)]\mid$. Wilf's conjecture states that $f(S)+1\leq n(S)\nu(S)$. We give a brief summary of knwon results on this problem. An equivalent formulation of the conjecture is presented, and it is used to prove our main result: semigroups whose embedding dimension is at least a half the mutliplicity satisfy Wilf's conjecture. We also show that the conjecture is verified by semigroups with small multiplicity and by those generated by a generalized arithmetic sequence.

Klara Stokes *Universitat Rovira i Virgili*

Numerical semigroups from combinatorial configurations

A (ν,b,d)-configuration is an incidence structure with ν points and b lines, such that for $d=\nu r/k=bk/r$ there are k points on every line, r lines through every point, two different lines intersect each other at most once and two different points are connected by a line at most once. Then d is always a natural number and it can be proved that the set of natural numbers defined in this way has the structure of a numerical semigroup. A triangle in a combinatorial configuration is a set of three points, pairwise collinear on different lines. It can be proved that the set of natural numbers d associated to the triangle-free combinatorial configurations also has the structure of a numerical semigroup. I will discuss what we know about the structure of these numerical semigroups.

Grazia Tamone *Universitá di Genova*

On some Weierstrass semigroups and the order bound

The classical theory of Weierstrass semigroups and the algebraic geometric (AG) codes are strictly related: given a smooth projective curve X over a field k and a k-rational point P of X , to the pair (X,P) one can associate a family of one-point algebraic geometric codes $\mathcal{C}_i$ and a numerical semigroup, called Weierstrass. The knowledge of this semigroup is useful to find the *order bound* $d_{ORD}(\mathcal{C}_i)$, which is a good bound for the related codes $\mathcal{C}_i$ if i is large enough. For this reason there is a new interest in identifying semigroups which are Weierstrass.
Not every semigroup has this property: counter-examples has been given by Buchweitz and other authors. As proved in Pinkham's thesis : if k is an algebraically closed field of characteristic 0,a semigroup S is Weierstrass if and only if the curve $X=\mathrm{Spec}\ (k[S])$ is smoothable, namely there exists a deformation of X with smooth fibres.
Here we prove that the semigroups generated by particular sequences are Weierstrass, by showing the associated monomial curves are smoothable. Further, we evaluate the order bound for the semigroups generated by arithmetic sequences.
Joint work with A. Oneto.

Albert Vico-Oton *Universitat Rovira i Virgili*

Non-homogeneous patterns on numerical semigroups

Arf numerical semigroups appear in many theoretical problems in algebraic geometry as well as in some applied areas such as coding theory [1, 7, 6, 5, 2, 3]. Arf semigroups are those semigroups such that for any elements x_1,x_2,x_3 in the semigroup with $x_1\geq x_2\geq x_3$, the integer $x_1+x_2-x_3$ also belongs to the semigroup.
This definition inspired studying the so-called patterns on numerical semigroups [4]. Patterns on numerical semigroups are multivariate polynomials such that evaluated at any decreasing sequence of elements of the semigroup give integers belonging to the semigroup.
For their simplicity, and for their inspiration in Arf semigroups, patterns were first defined to be linear and homogeneous. However, other families of numerical semigroups have appeared lately in very different areas of applied mathematics which satisfy linear non-homogeneous patterns. This suggests the need for studying non-homogeneous patterns on numerical semigroups.
We will give some motivating examples and some results on non-homogeneous linear patterns.
This is joint work with M. Bras-Amorós

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